

ATOMIC PHYSICS AND RADIATION

Energies and temperatures are in eV; all other units are cgs except where noted. Z is the charge state ($Z = 0$ refers to a neutral atom); the subscript e labels electrons. N refers to number density, n to principal quantum number. Asterisk superscripts on level population densities denote local thermodynamic equilibrium (LTE) values. Thus N_n^* is the LTE number density of atoms (or ions) in level n .

Characteristic atomic collision cross section:

$$(1) \quad \pi a_0^2 = 8.80 \times 10^{-17} \text{ cm}^2.$$

Binding energy of outer electron in level labelled by quantum numbers n, l :

$$(2) \quad E_\infty^Z(n, l) = -\frac{Z^2 E_\infty^H}{(n - \Delta_l)^2},$$

where $E_\infty^H = 13.6 \text{ eV}$ is the hydrogen ionization energy and $\Delta_l = 0.75l^{-5}$, $l \gtrsim 5$, is the quantum defect.

Excitation and Decay

Cross section (Bethe approximation) for electron excitation by dipole allowed transition $m \rightarrow n$ (Refs. 32, 33):

$$(3) \quad \sigma_{mn} = 2.36 \times 10^{-13} \frac{f_{mn} g(n, m)}{\epsilon \Delta E_{nm}} \text{ cm}^2,$$

where f_{mn} is the oscillator strength, $g(n, m)$ is the Gaunt factor, ϵ is the incident electron energy, and $\Delta E_{nm} = E_n - E_m$.

Electron excitation rate averaged over Maxwellian velocity distribution, $X_{mn} = N_e \langle \sigma_{mn} v \rangle$ (Refs. 34, 35):

$$(4) \quad X_{mn} = 1.6 \times 10^{-5} \frac{f_{mn} \langle g(n, m) \rangle N_e}{\Delta E_{nm} T_e^{1/2}} \exp\left(-\frac{\Delta E_{nm}}{T_e}\right) \text{ sec}^{-1},$$

where $\langle g(n, m) \rangle$ denotes the thermal averaged Gaunt factor (generally ~ 1 for atoms, ~ 0.2 for ions).

Rate for electron collisional deexcitation:

$$(5) \quad Y_{nm} = (N_m^*/N_n^*)X_{mn}.$$

Here $N_m^*/N_n^* = (g_m/g_n) \exp(\Delta E_{nm}/T_e)$ is the Boltzmann relation for level population densities, where g_n is the statistical weight of level n .

Rate for spontaneous decay $n \rightarrow m$ (Einstein A coefficient)³⁴

$$(6) \quad A_{nm} = 4.3 \times 10^7 (g_m/g_n) f_{mn} (\Delta E_{nm})^2 \text{ sec}^{-1}.$$

Intensity emitted per unit volume from the transition $n \rightarrow m$ in an optically thin plasma:

$$(7) \quad I_{nm} = 1.6 \times 10^{-19} A_{nm} N_n \Delta E_{nm} \text{ watt/cm}^3.$$

Condition for steady state in a corona model:

$$(8) \quad N_0 N_e \langle \sigma_{0n} v \rangle = N_n A_{n0},$$

where the ground state is labelled by a zero subscript.

Hence for a transition $n \rightarrow m$ in ions, where $\langle g(n, 0) \rangle \approx 0.2$,

$$(9) \quad I_{nm} = 5.1 \times 10^{-25} \frac{f_{nm} g_m N_e N_0}{g_0 T_e^{1/2}} \left(\frac{\Delta E_{nm}}{\Delta E_{n0}} \right)^3 \exp \left(-\frac{\Delta E_{n0}}{T_e} \right) \frac{\text{watt}}{\text{cm}^3}.$$

Ionization and Recombination

In a general time-dependent situation the number density of the charge state Z satisfies

$$(10) \quad \frac{dN(Z)}{dt} = N_e \left[-S(Z)N(Z) - \alpha(Z)N(Z) \right. \\ \left. + S(Z-1)N(Z-1) + \alpha(Z+1)N(Z+1) \right].$$

Here $S(oZ)$ is the ionization rate. The recombination rate $\alpha(Z)$ has the form $\alpha(Z) = \alpha_r(Z) + N_e \alpha_3(Z)$, where α_r and α_3 are the radiative and three-body recombination rates, respectively.

Classical ionization cross-section³⁶ for any atomic shell j

$$(11) \quad \sigma_i = 6 \times 10^{-14} b_j g_j(x) / U_j^2 \text{ cm}^2.$$

Here b_j is the number of shell electrons; U_j is the binding energy of the ejected electron; $x = \epsilon/U_j$, where ϵ is the incident electron energy; and g is a universal function with a minimum value $g_{\min} \approx 0.2$ at $x \approx 4$.

Ionization from ion ground state, averaged over Maxwellian electron distribution, for $0.02 \lesssim T_e/E_\infty^Z \lesssim 100$ (Ref. 35):

$$(12) \quad S(Z) = 10^{-5} \frac{(T_e/E_\infty^Z)^{1/2}}{(E_\infty^Z)^{3/2} (6.0 + T_e/E_\infty^Z)} \exp\left(-\frac{E_\infty^Z}{T_e}\right) \text{ cm}^3/\text{sec},$$

where E_∞^Z is the ionization energy.

Electron-ion radiative recombination rate ($e + N(Z) \rightarrow N(Z-1) + h\nu$) for $T_e/Z^2 \lesssim 400$ eV (Ref. 37):

$$(13) \quad \alpha_r(Z) = 5.2 \times 10^{-14} Z \left(\frac{E_\infty^Z}{T_e}\right)^{1/2} \left[0.43 + \frac{1}{2} \ln(E_\infty^Z/T_e) \right. \\ \left. + 0.469(E_\infty^Z/T_e)^{-1/3} \right] \text{ cm}^3/\text{sec}.$$

For $1 \text{ eV} < T_e/Z^2 < 15 \text{ eV}$, this becomes approximately³⁵

$$(14) \quad \alpha_r(Z) = 2.7 \times 10^{-13} Z^2 T_e^{-1/2} \text{ cm}^3/\text{sec}.$$

Collisional (three-body) recombination rate for singly ionized plasma:³⁸

$$(15) \quad \alpha_3 = 8.75 \times 10^{-27} T_e^{-4.5} \text{ cm}^6/\text{sec}.$$

Photoionization cross section for ions in level n, l (short-wavelength limit):

$$(16) \quad \sigma_{\text{ph}}(n, l) = 1.64 \times 10^{-16} Z^5 / n^3 K^{7+2l} \text{ cm}^2,$$

where K is the wavenumber in Rydbergs (1 Rydberg = $1.0974 \times 10^5 \text{ cm}^{-1}$).

Ionization Equilibrium Models

Saha equilibrium:³⁹

$$(17) \quad \frac{N_e N_1^*(Z)}{N_n^*(Z-1)} = 6.0 \times 10^{21} \frac{g_1^Z T_e^{3/2}}{g_n^{Z-1}} \exp\left(-\frac{E_\infty^Z(n,l)}{T_e}\right) \text{ cm}^{-3},$$

where g_n^Z is the statistical weight for level n of charge state Z and $E_\infty^Z(n,l)$ is the ionization energy of the neutral atom initially in level (n,l) , given by Eq. (2).

In a steady state at high electron density,

$$(18) \quad \frac{N_e N^*(Z)}{N^*(Z-1)} = \frac{S(Z-1)}{\alpha_3},$$

a function only of T .

Conditions for LTE:³⁹

(a) Collisional and radiative excitation rates for a level n must satisfy

$$(19) \quad Y_{nm} \gtrsim 10A_{nm}.$$

(b) Electron density must satisfy

$$(20) \quad N_e \gtrsim 7 \times 10^{18} Z^7 n^{-17/2} (T/E_\infty^Z)^{1/2} \text{ cm}^{-3}.$$

Steady state condition in corona model:

$$(21) \quad \frac{N(Z-1)}{N(Z)} = \frac{\alpha_r}{S(Z-1)}.$$

Corona model is applicable if⁴⁰

$$(22) \quad 10^{12} t_I^{-1} < N_e < 10^{16} T_e^{7/2} \text{ cm}^{-3},$$

where t_I is the ionization time.

Radiation

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Average radiative decay rate of a state with principal quantum number n is

$$(23) \quad A_n = \sum_{m < n} A_{nm} = 1.6 \times 10^{10} Z^4 n^{-9/2} \text{ sec.}$$

Natural linewidth (ΔE in eV):

$$(24) \quad \Delta E \Delta t = h = 4.14 \times 10^{-15} \text{ eV sec,}$$

where Δt is the lifetime of the line.

Doppler width:

$$(25) \quad \Delta\lambda/\lambda = 7.7 \times 10^{-5} (T/\mu)^{1/2},$$

where μ is the mass of the emitting atom or ion scaled by the proton mass.

Optical depth for a Doppler-broadened line:³⁹

$$(26) \quad \tau = 3.52 \times 10^{-13} f_{nm} \lambda (Mc^2/kT)^{1/2} NL = 5.4 \times 10^{-9} f_{mn} \lambda (\mu/T)^{1/2} NL,$$

where f_{nm} is the absorption oscillator strength, λ is the wavelength, and L is the physical depth of the plasma; M , N , and T are the mass, number density, and temperature of the absorber; μ is M divided by the proton mass. Optically thin means $\tau < 1$.

Resonance absorption cross section at center of line:

$$(27) \quad \sigma_{\lambda=\lambda_c} = 5.6 \times 10^{-13} \lambda^2 / \Delta\lambda \text{ cm}^2.$$

Wien displacement law (wavelength of maximum black-body emission):

$$(28) \quad \lambda_{\text{max}} = 2.50 \times 10^{-5} T^{-1} \text{ cm.}$$

Radiation from the surface of a black body at temperature T :

$$(29) \quad W = 1.03 \times 10^5 T^4 \text{ watt/cm}^2.$$

Bremsstrahlung from hydrogen-like plasma:²⁶

$$(30) \quad P_{\text{Br}} = 1.69 \times 10^{-32} N_e T_e^{1/2} \sum \left[Z^2 N(Z) \right] \text{ watt/cm}^3,$$

where the sum is over all ionization states Z .

Bremsstrahlung optical depth:⁴¹

$$(31) \quad \tau = 5.0 \times 10^{-38} N_e N_i Z^2 \bar{g} L T^{-7/2},$$

where $\bar{g} \approx 1.2$ is an average Gaunt factor and L is the physical path length.

Inverse bremsstrahlung absorption coefficient⁴² for radiation of angular frequency ω :

$$(32) \quad \kappa = 3.1 \times 10^{-7} Z n_e^2 \ln \Lambda T^{-3/2} \omega^{-2} (1 - \omega_p^2/\omega^2)^{-1/2} \text{ cm}^{-1};$$

here Λ is the electron thermal velocity divided by V , where V is the larger of ω and ω_p multiplied by the larger of $Z e^2/kT$ and $\hbar/(mkT)^{1/2}$.

Recombination (free-bound) radiation:

$$(33) \quad P_r = 1.69 \times 10^{-32} N_e T_e^{1/2} \sum \left[Z^2 N(Z) \left(\frac{E_\infty^{Z-1}}{T_e} \right) \right] \text{ watt/cm}^3.$$

Cyclotron radiation²⁶ in magnetic field \mathbf{B} :

$$(34) \quad P_c = 6.21 \times 10^{-28} B^2 N_e T_e \text{ watt/cm}^3.$$

For $N_e kT_e = N_i kT_i = B^2/16\pi$ ($\beta = 1$, isothermal plasma),²⁶

$$(35) \quad P_c = 5.00 \times 10^{-38} N_e^2 T_e^2 \text{ watt/cm}^3.$$

Cyclotron radiation energy loss e -folding time for a single electron:⁴¹

$$(36) \quad t_c \approx \frac{9.0 \times 10^8 B^{-2}}{2.5 + \gamma} \text{ sec},$$

where γ is the kinetic plus rest energy divided by the rest energy mc^2 .

Number of cyclotron harmonics⁴¹ trapped in a medium of finite depth L :

$$(37) \quad m_{\text{tr}} = (57\beta BL)^{1/6},$$

where $\beta = 8\pi NkT/B^2$.

Line radiation is given by summing Eq. (9) over all species in the plasma.